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Research article

MATHEMATICAL MODEL TO MONITOR SEEPAGE AND DISCHARGE VELOCITY INFLUENCED BY VOLUME OF VOID IN COASTAL AREA OF PORT HARCOURT

Eluozo, S. N.

Subaka Nigeria Limited Port Harcourt Rivers State of Nigeria
Director and principal consultant Civil and Environmental Engineering, Research and Development
E-mail: Soloeluozo2013@hotmail.com
E-mail: solomoneluozo2000@yahoo.com

Abstract

Seepage discharge in soil and water environment are influenced several factors in the formation, these are base on the stratification of the formation, the rate of seepage are expressed from percentage of void ratio deposition in the strata, the seepage discharge are determined by the formation characteristic under the influences of geological formation, the seepage challenges affect several areas, the seepage discharge develop serious challenges in foundation design and construction due to displacement of water between the intercedes of the soil, the principle of Darcy law has for several decade play lots of roles in the flow principle of fluid between the soil strata, but the establish model will monitor the rate of seepage discharge in soil and water environment, the rate of discharge through seepage will be examined in any design of foundation, including mentoring pollution transport in phreatic deposition or confined bed, experts in soil and water engineering can applied the model to monitor seepage discharge of any sources to prevent construction failure or prevent pollution transport in coastal environment that deposition deltaic formations.

Keywords: mathematical model seepage velocity, and volume of void

Introduction

This scaling law for seepage velocity has been accepted and commonly used, but the question of whether it is the Darcy's permeability (hydraulic conductivity) or the hydraulic gradient that is a function of gravity has not been addressed properly. This issue was highlighted by Goodings (1979), who points out to the multiplicity of the concepts in scaling flow velocity. Butterfield (2000) and Dean (2001) also discussed this issue. Pokrovsky and Fyodorov (1968), Cargill and Ko (1983) , Tan and Scott (1985) and more recently Singh and Gupta (2000) are among many others who have considered permeability (k) to be directly proportional to gravity and hydraulic gradient (i) to be independent of gravity. While this explains why seepage velocity has a scaling law of N ($m\ p\ v = N\ v$), there is an alternative explanation for the increase of seepage velocity in a centrifuge. Schofield (1980), Hussaini et al (1981), Goodings (1984), and Taylor (1987) have all suggested that permeability to be independent of gravity and it is the hydraulic gradient which has got a scaling factor of N. Since both sides of the explanation result in the same final answer $m\ p\ v = N\ v$ and it is the final seepage velocity that is considered important in many cases, the controversy has often been overlooked. In this technical note we attempt to resolve this controversy by using the energy gradient as the driving force on the pore fluid. Soil hydraulic properties like the retention capacity and hydraulic as well as gas conductivity have agronomical and ecological implications. Drainage, evaporation and water-uptake by roots are just few examples where the knowledge about the rate of water flow through the soil plays an important role (Plagge *et al*., 1990; Kutilek and Nielsen, 1994). The relative proportion of the three phases (water, gas and solid) of the soil is influenced by properties like texture, structure, biological activity, weather and soil management (Hillel, 1998). In these terms the porous media can be characterized in their volume and function which is of great relevance to understand processes related to water, air and heat transport in soils (Oschner *et al*., 2001; Dörner and Horn, 2006). Soil volumes are affected by mechanical stresses (e.g. tillage-induced soil compaction, Blackwell *et al*., 1986; Horn et al., 1991; Horn et al., 1995; Ball *et al*., 1997; McNabb *et al*, 2001) and internal forces (e.g. wetting and drying cycles, Peng and Horn, 2007; Bartoli *et al*., 2007). The magnitude of volume changes are controlled by the mechanical stability of the soil or by the level of the actual in comparison with previous internal stresses. The internal stresses in soils containing more than 12% of clay induce aggregate formation, which results firstly in a prismatic structure with a dominant vertical pore function orientation. Repeated swelling and shrinkage creates tensile and shear induced crack formation in blocky and thereafter in a subangular blocky structure (Horn and Smucker, 2005). The effects of soil compaction on soil structure have been investigated by many authors (Blackwell *et al*., 1986; Horn *et al*., 1995; Ball *et al*., 1997; McNabb *et al*., 2001). It is also well known that soils are able to shrink and swell (Peng *et al*., 2006). However, with respect to the calculation of hydraulic properties, it is always assumed that soils behave like a rigid body.

2. Theoretical Background

The volumes of void in soil are depending on some factors, these are base on the structural stratification of soil under the influences of geologic process by which soil is formed. The description limits are through size of the soil particles, including mechanical analysis of soil. But in natural depositions soil are in three phase system consisting of soil solid, water and air. The element of soil of volume and weight are proportional to each other. The volume relationships commonly used for the three phase in a soil element are void, porosity, and degree of saturation, void ratio is defined as the volume of void too volume of solid. With the development of social economy, more and more solid waste is produced by urban residents and industry which affects environment seriously. To improve the surrounding environment, people plant trees and reclaim landfill. But there are still many accidents about the pollutant water which lead to drinking water difficulty, reduction of crop output and toxicosis of human and livestock at surrounding areas. The landfill leachate is one of the most important factors of environmental pollution. Contaminant transport of landfill leachate is affected by advection, dispersion, sorption, and decay of biology. Therefore the problem of environment pollution by waste becomes hot to environment and water resource scholar in developed countries. Study of contaminant transport predicts and pollute-control in polluted landfill site just began in recent years in some developed nations Permeability, as the name implies (ability to permeate), is a measure of how easily a fluid can flow through a porous medium. In geotechnical engineering, the porous medium is soils and the fluid is water at ambient temperature. Generally, coarser the soil grains, larger the voids and larger the permeability. Therefore, gravels are more permeable than silts. Hydraulic conductivity is another term used for permeability, often in environmental engineering studies. Flow of water through soils is called seepage. Whenever there is seepage (e.g., beneath a concrete dam or a sheet pile), it is often necessary to estimate the quantity of the seepage, and permeability becomes the main parameter here. When water flows through soils, whether beneath a concrete dam or a sheet pile, the seepage velocity is often very small When water flows through soils, from *upstream* to *downstream*, due to difference in water level as in some energy is lost in overcoming the resistance provided by the soils. This loss of energy, expressed as total head loss (h_L), is simply the difference in water levels. The pressure *p* is the pore water pressure (*u*), and therefore pore water pressure at any point Seepage takes place through the sub soil, due to head difference between upstream and downstream water levels. If we know the permeability of the soil, how do we calculate the discharge through the soil? The seepage of underground water affects the stress field through the change of seepage volume force. And variation of stress field changes the pore characteristics of slope medium. Then it changes the seepage field. The coupling of fluid and solid is the internal cause of slope failure. This paper based on the internal mechanism of slope failure, and established mathematical model of slope failure system under fluid-solid coupling, calculated slope stability coefficient by use of strength reduction FEM, and analyzed the effect of slope stability considering seepage action. This condition has been stress in the concept of seepage discharge as it expressed mathematically below.

3. Governing Equation

$$\frac{Av + As}{Av}\frac{\partial Vs}{\partial L} - \frac{\partial Vs}{\partial t} + \frac{(Av + As)}{AvL} + \frac{\partial Vs}{\partial L} V \frac{(Vv + Vs)}{Vv} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

$$\frac{\partial Vs}{\partial L} = S^1Vs(L) - SVs(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial Vs}{\partial t} = S^1Vs(L) - SVs(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{\partial Vs}{\partial L} = S^1Vs(L) - SVs(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$S^1Vs(L) - \frac{Av + As}{Av}\left[S^1(L) - S^1Vs(o)\right] + V\frac{(Av + As)}{AvL}\left[S^1Vs(t) - S^1Vs(z)\right] \quad \ldots\ldots \quad (5)$$

$$S^1(L) - Vs(o) = \frac{Av + As}{Av}S^1Vs(o) - \frac{(Av + As)}{AvL}Vs(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$V\frac{(Av + As)}{AvL}Vs(L) - V\frac{(Av + As)}{AvL}SVs(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$V\frac{(Vv + Vs)}{Vv}Vs(z) - SVs(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Let $Vs(o) = 0$

We have

$$S^1(L) - \frac{(Av + As)}{Av}Vs(o) + V\frac{(Av + As)}{AvL}Vs(t) + V\frac{(Vv + Vs)}{Vv}S^1Vs(L) \quad \ldots\ldots \quad (9)$$

$$VS^1(L) = \frac{1}{S}\left[\frac{(Av + As)}{Av}Vs(L) - V\frac{(Av + As)}{AvL}S^1 + V\frac{(Vv + Vs)}{Vv}S^1Vs(L)\right] \quad \ldots\ldots \quad (10)$$

$$VS(L) = \frac{1}{S^1}\left[\frac{(Av + As)}{Av}Vs(L) - V\frac{(Av + As)}{AvL}S^1 + V\frac{(Vv + Vs)}{Vv}\right] \quad \ldots\ldots\ldots\ldots \quad (11)$$

$$VS^1(L) = \frac{\frac{(Av + As)}{Av}Vs(L) - V\frac{(Av + As)}{AvL}Vs(t) + V\frac{(Vv + Vs)}{Vv}}{S^1} \quad \ldots\ldots\ldots\ldots \quad (12)$$

$$VS(L) = \frac{(Av + As)}{Av}S^1Vs(L) + V\frac{(Vv + Vs)}{Vv}VS(t) \quad \ldots\ldots\ldots\ldots \quad (13)$$

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$$VS(L) = \frac{\frac{(Av + As)}{Av} Vs(L) \; = V \frac{(Av + As)}{AvL} Vs(L) + V \frac{(Vv + Vs)}{Vv}}{S} \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$VS(L) = \left[\frac{(Av + As)}{Av} + V \frac{(Av + As)}{AvL} + V \frac{(Vv + Vs)S^1}{Vs} \right] VS(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

$$S^1 VS(L) = \left(\frac{(Av + As)}{Av} + V \frac{(Av + As)}{AvL} + V \frac{(Vv + Vs)S^1}{Vs} \right) VS(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$\frac{Av + As}{Av} + V \frac{(Av + As)}{AvL} + V \frac{(Vv + Vs)S^1}{Vs} \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

$$VS(L) \frac{S^1(L)}{\frac{Av + As}{Av} + V \frac{(Av + As)}{AvL} + V \frac{(Vv + Vs)S^1}{Vs}} \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

Furthermore, considering the boundary condition, we have the following

$$\text{At } t = 0 \qquad V^1 S(o) = VS(o) \; = \; 0$$

$$\frac{Av + As}{Av} VS(L) \; - V \frac{(Av + As)S^1}{AvL} VS(L) \; + V \frac{(Vv + Vs)}{Vv} VS(t) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

$$VS(L) \; = \; \frac{Av + As}{Av} - V \frac{(Av + As)}{AvL} + V \frac{(Vv + Vs)}{Vs} \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

Considering the following boundary conditions when

$$\text{At } t > 0 \qquad V^1 S(o) = VS(o)$$

Applying the boundary condition into this equation

$$\frac{Av + As}{Av} VS(L) - \frac{(Av + As)}{Av} + V \frac{(Av + As)}{AvL} VS(L) \; + V \frac{(Av + As)}{AvL} VSo -$$

$$S(L) + V \frac{(Vv + Vs)}{Vv} VS(t) \; + V \frac{(Vv + Vs)}{Vv} VSo + S(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

$$\frac{(Av + As)}{Av}(L) - \frac{(Av + As)}{AvL} VS(o) \; \frac{(Av + As)}{Av} VSo \; + V \frac{(Vv + Vs)}{Vv} VSo \quad \ldots \quad (22)$$

$$VS(L) = \left[\frac{Av + Ass}{Av} = V\frac{(Av + As)}{Av} + V\frac{(Av + As)}{AvL} + V\frac{(Vv + Vs)}{Vv}\right]VSo \quad \ldots \qquad (23)$$

$$VS(L) = \frac{Av + Ass}{Av} - \frac{(Av + As)}{Av} - V\frac{(Av + As)}{AvL} + V\frac{(Vv + Vs)}{Vv}VSo \quad \ldots\ldots\ldots \qquad (24)$$

$$VS(L) = \frac{\frac{(Av + Ass)}{Av} - \frac{(Av + As)}{Av} - V\frac{(Av + As)}{AvL} + V\frac{(Vv + Vs)}{Vv}VSo}{\frac{(Av + As)}{Av}S - \frac{(Av + As)}{Av} + V\frac{(Vv + Vs)}{Vv}} \quad \ldots\ldots\ldots \qquad (25)$$

Applying quadratic equation to determine denominator for the equation

$$\frac{Av + Ass}{Av} - V\frac{(Av + As)}{AvL} + V\frac{(Vv + Vs)}{Vv} = 0 \quad \ldots\ldots\ldots \qquad (26)$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (27)$$

Where $a = \frac{Av + As}{Av}$, $b = V\frac{(Av + As)}{AvL}$ and $c = V\frac{(Vv + Vs)}{Vv}VS$

For simplicity denoting the expressed functions parameter of the following

Let $\frac{Av + As}{Av} = Q$, $V\frac{(Av + As)}{AvL} = \lambda^2$ and $V\frac{(Vv + Vs)}{Vv} = \alpha$

Integrating the express parameters into the quadratic function we have:

$$S = \frac{-\lambda^2 \pm \sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (28)$$

$$\left[S_1 = \frac{\lambda^2 + \sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]\left[S_2 = \frac{\lambda^2 - \sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]$$

$$\ell^{\left[\frac{\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t\left[\frac{\lambda\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (29)$$

The inverse Laplace of the equation yield

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$$VS(L) = \left[\frac{Q}{t} + Q + \lambda + \alpha\right] VSo\ \ell^{\left[\frac{\lambda + \sqrt{\lambda^2 + 4Q\alpha VS}}{2QVS}\right]t}$$

$$\left[\left[\frac{\lambda - \sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]\right]t - \left[\frac{\lambda - \sqrt{\lambda^2 + 4Q\alpha VS}}{2QVS}\right]t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

$$VS(L) = \left[\frac{\lambda}{t^2} VSo\right]\left[\left[\frac{\lambda + \sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]\right] \ell^{\left[\frac{\lambda\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t}$$

$$\ell^{\left[\frac{\lambda\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t - \left[\frac{\lambda\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

At this point $VSo = 0 \quad t \neq 0$

For equation (30) at $t = 0\ VS(o) = VSo,\ we\ \ have$

$$VSo = \left[(Q + \lambda + \alpha)\, VSo\ \ (1+1+1)\right] = 0 = (Q + \lambda + \alpha)$$

$$Hence\ \ Q + \lambda + \alpha = 0$$

Equation (31) becomes

$$VS(L) - VSo\left[\frac{\lambda}{t^2} + 2\right]\left[\frac{\lambda + \sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t\ \left[\frac{\lambda\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right] \quad \ldots\ldots\ldots \quad (32)$$

We recall that $Hence\ \ \ell^x + \ell^{-x} = 2Cos\,x,$ so that equation (32) can be expressed as:

$$VS(L) - [\lambda + 2]VSo\ Cos\left[\frac{\lambda\sqrt{\lambda^2 - 4Q\alpha VS}}{2QVS}\right]t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

The expression in [33] is the final model to monitor the seepage discharge and velocities influenced by volume of void, this condition are determined by the percentage of void ratio in the formation. The structure of the soil in most case determined the percentage of several variation from void are deposited due geological setting of the formation, the developed model from the governing equation were able to express numerous condition that establish the structure of the seepage discharge, the velocity that express the charge at any direction of flow are determine by the flow path either longitudinal tortuosity deposited the strata. The expressed model were able establish this condition in like manner with the principle of Darcy law equation.

4. Conclusion

The seepage discharge from volume of void are express to monitor variations of soil through higher percentage of void ratio, the formation in coastal area develop high degree of void, this implies that the structural stratification are influenced by the coastal condition under the influences of environmental condition, the formation in design of any impose load like water tank need serious attention to the soil bearing capacity, although this instruction govern every design in foundation of any impose load, but much more attention should be given to coastal environment due to several formation variation, this include other influences in the coastal area that will normally developed more challenges in design and construction of foundation in terms seepage discharge, this will definitely developed variation of water displacement and as well as serious challenges to the foundation design and construction. The developed model establish in general should be applied in any condition of seepage discharge in coastal environment.

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